Mathematics 300 Spring 1999 Exam 1

February 23

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Name

Technology used: ____

Textbook/Notes used: ____

The beginning of wisdom is the definition of terms. – **Socrates**

It is a safe rule to apply that, when a mathematical or philosophical author writes with a misty profundity, he is talking nonsense. – **A.N. Whitehead**

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

The Problems

I. Do any one of the following

1. Use a truth table to prove that $(\sim q \wedge (p \Longrightarrow q)) \Longrightarrow \sim p$ is always true.
2. Negate the following logical statement

$$\forall \varepsilon \; \exists \delta \; \forall x \; \left(|x-a| < \delta \Longrightarrow |f(x) - f(a)| < \varepsilon \right).$$

II Do one of the following.

1. Using any result through Chapter 2 of the text and any exercise up to and including Major Exercise 7 part (b), prove the following. In a finite projective plane $\mathcal{M}$ in which every point has exactly $n+1$ distinct lines incident with it, there are exactly n^2+n+1 distinct lines.
2. What is the smallest number of lines possible in a model of incidence geometry in which there are exactly 5 points? Include a careful argument supporting your claim (but you need not provide a formal proof.)
3. Using any result up to and including Major exercise 7 of Chapter 2 of Greenberg, do part (b) of Major exercise 8. That is, let $\mathcal{A}$ be a finite affine plane (in which, by Major exercise 2, every line has exactly the same number of distinct points. Call this number n. Suppose, in addition, we know that each point in $\mathcal{A}$ has exactly $n+1$ lines passing through it. Prove the total number of points in A is n^2.

III Do any two of the following.

1. Using the Same Side and Opposite Side lemmas and any result up to and including Proposition 3.2, prove that if $A*B*C$ and $A*C*D$, then the four points A, B, C, and D are distinct and collinear.
2. Using any result up to and including Pasch's Theorem, prove the last part of Proposition 3.5: Given $A*B*C$. Show that B is the only point common to segments AB and BC. [You may use the fact that $AC = AB \cup BC$.]

3. Using any result up to and including part (a) of Proposition 3.9, prove the following. If D is a point interior to triangle ΔABC, then any ray emanating from D must intersect the triangle.

IV Do any two of the following.

1. Using any previous result, prove Proposition 3.11. If $A * B * C$, $D * E * F$, $AB \cong DE$, and $AC \cong DF$, then $BC \cong EF$.
2. Using any previous result, prove part (d) of Proposition 3.13. If $AB < CD$, and $CD < EF$, then $AB < EF$.
3. Using any result up to and including part (b) of Proposition 3.8, prove that if D is in the interior of angle $\measuredangle CAB$ and $C * A * E$, then segment BE does not intersect the ray opposite $\overrightarrow{AD}$.
4. Using any previous result, prove Proposition 3.17 (the Angle-Side-Angle criterion for congruence of triangles). Given $\triangle ABC$ and $\triangle DEF$ with $\measuredangle A \cong \measuredangle D$, $\measuredangle C \cong \measuredangle F$, and $AC \cong DF$, then $\triangle ABC \cong \triangle DEF$